





# Discrete time Markov models of cognitive transitions: Assessing goodness of fit

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## Supplementary materials

### Multinomial Representation of Discrete-Time Markov Models

In discrete-time settings, transition probabilities between states can be estimated using a multinomial logistic regression framework. Let  $Y_{i,t}$  denote the state occupied by individual  $i$  at time  $t$ , where  $Y_{i,t} \in \{1, 2, \dots, K\}$ . The probability of transitioning from state  $j$  at time  $t$  to state  $k$  at time  $t + 1$  can be expressed as:

$$p_{jk}(t, t + 1) = P(Y_{i,t+1} = k | Y_{i,t} = j).$$

When covariates  $\mathbf{z}_{i,t}$  are included these probabilities can be modelled using a multinomial logistic regression formulation. For non-reference categories  $k = 2, \dots, K$ , the linear predictor is

$$\eta_{i,t}^{(k)} = \log\left(\frac{p_{jk}(\mathbf{z}_{i,t})}{p_{j1}(\mathbf{z}_{i,t})}\right) = \alpha_{jk} + \mathbf{z}_{i,t}^\top \boldsymbol{\beta}_{jk} \quad \text{for } k = 2, \dots, K. \quad (1)$$

Within the formulation,  $p_{j1}$  is the reference probability (e.g., remaining in state  $j$ ),  $\alpha_{jk}$  is a state-pair specific intercept, and  $\boldsymbol{\beta}_{jk}$  is a vector of coefficients for covariates  $\mathbf{z}_{i,t}$  (of which  $Y_{i,t}$  is inclusive).

The resulting transition probabilities are obtained from the multinomial link function such that the transition probabilities from state  $j$  are given by:

$$p_{jk}(\mathbf{z}_{i,t}) = \frac{\exp(\eta_{i,t}^{(k)})}{1 + \sum_{k=2}^K \exp(\eta_{i,t}^{(k)})}, \quad \text{for } k = 2, \dots, K \quad (2)$$

and for the reference category,

$$p_{j1}(\mathbf{z}_{i,t}) = \frac{1}{1 + \sum_{k=2}^K \exp(\eta_{i,t}^{(k)})}. \quad (3)$$

This formulation provides a convenient way to estimate covariate-dependent transition probabilities in discrete-time Markov models using standard multinomial regression software.

## References